Shear Langmuir vortex: An elementary mode of plasma collective behavior

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Linear evolution of electrostatic perturbations in a cold, unmagnetized, two-component plasma with *shear* flow is studied. It is shown that the velocity shear induces, due to the non-normality of the linear dynamics, a new elementary mode of plasma *nonperiodic* collective behavior—the ''shear Langmuir vortex.'' The mode, characterized by intense energy exchange with the mean flow, is associated with nonoscillatory motion of the plasma species. In the low-shear limit, the vortex patterns can be described by extremely simple analytic solutions both for the two-dimensional and three-dimensional cases. It is shown that the two-dimensional vortices are able to extract the mean flow energy only *transiently*, while the three-dimensional structures can grow *asymptotically*. In moderate and high shear regimes, the conversion of shear Langmuir vortexes into Langmuir oscillations becomes possible; the latter has nonresonant character and happens abruptly in time. [S1063-651X(98)05706-7]

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I. INTRODUCTION

The study of vortices and vortex motion, with deep roots in the classical works of Helmholtz, Lord Kelvin, Prandtl, and his Gottingen school, is becoming increasingly more relevant for a vast number of problems arising in physics, astrophysics, engineering, and mathematics. Being a field of active research in many different disciplines (including such important areas as the theory of coherent structures in fluid and plasma turbulence, theory of chaotic motion and dynamical systems, theory of line vortices and vortex rings in liquid helium, etc.), vortex dynamics is currently recognized as a universal and exciting *interdisciplinary* scientific problem [1].

The vortical and the wave modes of motion are two generic classes of collective behavior that seem to pervade all nature. Hydrodynamic and plasma flows are examples of physical systems, where both these modes are present, and where they play an important role in the overall dynamics, linear and nonlinear. Vortical motions have been traditionally, studied in neutral fluids. It was only in the early 1980's that a certain degree of similarity between the vortices in plasmas and neutral fluids was noticed, and somewhat exploited [2,3]. A search for vortical motions in plasmas, however, was not quite vigorous because of their expected association with rather complicated, nonlinear systems. Contemporary progress in the study of *neutral fluid shear flows*, however, has revealed a bundle of interesting physical effects associated with the vortex and wave *linear* dynamics. It is hoped that the same methods can be effectively applied to the discovery and delineation of novel vortical modes in *plasma shear flows*.

Before taking a close look on this problem it should be stressed that the stability of shear flows remains a serious problem in fluid mechanics, and in plasma physics. Although the standard stability theory (the normal modes approach) has been very successful in dealing with a variety of shear flows, it does run into serious problems in some rather basic and important cases; the non-self-adjoint character of the governing equations [4–6] is, often, the source of trouble.

The normal mode approach, designed to examine asymptotic stability, is perfectly fine for self-adjoint operators with mutually orthogonal eigenfunctions. The eigenfunctions associated with non-self-adjoint differential equations are, however, not mutually orthogonal, and may strongly interfere to force an algebraic (nonexponential) behavior for early times [7]. Several recent investigations [4–6,8,9] (based on the original ideas due to Lord Kelvin [10] and Orr [11,12]) have revealed that a superposition of eventually decaying normal modes may grow initially and that this *transient* growth can be significant. Naturally an investigation of this important phenomenon is beyond the purview of the standard stability theory.

In practical terms, it means that in parallel shear flows, where the defining equations may display mathematical peculiarities of the type mentioned above, one should look very carefully for new modes and also for new phenomena in the established modes; it is likely that such aspects of collective motions may have been overlooked in the framework of traditional analysis. A convenient tool for such a survey is the so-called *nonmodal approach* (due to Kelvin [10]). In this approach, the initial value problem is solved by determining the temporal evolution of the *spatial Fourier harmonics* (SFH's) of a mode in a moving frame. This approach is particularly suited for tracing out the time history of nonexponentially evolving disturbances, and has already been applied to several important hydrodynamical [8,9,13–17], hydromagnetic and plasma [18–26] shear flows. Several unexpected and basic results, associated with the *linear dynamics* of disturbances, have followed in the wake.

In particular, the transient algebraically growing solutions, characterized by *vortical* motion of species, are found to be common for a wide range of shear flows: accretion shear flows [13], MHD shear flows of the standard [19], and the electron-positron [22] plasmas. In a recent work [26], it was shown that velocity shear induces the excitation of a completely *new class* of nonperiodic, electrostatic perturbations with vortical motion of the plasma ions. These perturbations, with their demonstrated ability for effective energy exchange with the mean flow, may play some or a *dominant* role in the linear dynamics of the system.

Another interesting feature of shear-induced vortices is their remarkable ability to convert into waves for moderate and high values of the shear. This phenomenon, discovered just recently [27], was demonstrated for a plane two-dimensional Couette flow of a neutral fluid but its rather general character encourages one to look for the existence of similar phenomena in plasma flows.

In the present article we demonstrate the existence of yet another mode of plasma vortical behavior for a *generic* two-component, cold, nonrelativistic plasma flow. The complexity of the vortex temporal evolution is studied for different values of the shear rate. The paper is arranged as follows:

In Sec. II, we develop the general formalism. We first derive the linearized electrostatic equations for the flow, and then implement the ''program'' for the nonmodal approach by deriving the ordinary differential equations (ODE's), describing the interplay of the conventional plasma features with the velocity shear. In Sec. III, we analyze the system (assuming zero mutual streaming of species) to derive a mode of collective motion: a nonperiodic, shear-induced plasma vortex.

The dynamics of these solutions, which can be called ''shear Langmuir vortexes'' (SLV), is further studied in the concluding Sec. IV. It is shown that the two-dimensional SLV are able to extract the background flow energy only *transiently*. However, in three dimensions (3D) the SLV can feed on the background flow energy even *asymptotically*. The criterion for the shear instability is derived and analyzed. It is further shown that for high enough values of the shear parameter, SLV solutions acquire noticeable wavelike features at large enough times. This phenomenon is described qualitatively as conversion of SLV's into plasma oscillations.

II. GENERAL FORMALISM

Consider a nonrelativistic, two-component, overall neutral, cold, unmagnetized fluid plasma, characterized by the charges q_s, densities n_s, and the laboratory-frame velocities $\vec{\mathcal{V}}_s$ (s is the species index). Let us further assume that the background mean velocities of the species are equally and linearly sheared:

$$\vec{\mathcal{V}}_s=\{\mathcal{V}_s+Ay,0\}. \tag{1}$$

Without loss of generality, we can take $\mathcal{V}_2>\mathcal{V}_1$. Neglecting the magnetic field produced by the streaming particles, the linear electrostatic response of this system is governed by the Poisson equation,

$$\Delta\varphi=-4\pi(q_1n_1+q_2n_2), \tag{2}$$

and the linearized fluid equations (motion and continuity) for each one of the species,

$$\mathcal{D}_sV_{s_x}+AV_{s_y}=-(q_s/m_s)\partial_x\varphi, \tag{3a}$$

$$\mathcal{D}_sV_{s_y}=-(q_s/m_s)\partial_y\varphi, \tag{3b}$$

$$\mathcal{D}_sV_{s_z}=-(q_s/m_s)\partial_z\varphi, \tag{3c}$$

$$\mathcal{D}_sn_s+N_s(\partial_xV_{s_x}+\partial_yV_{s_y}+\partial_zV_{s_z})=0, \tag{4}$$

where $\mathbf{V}_s$ is the fluctuating velocity, and $\mathcal{D}_s\equiv\partial_t+(\mathcal{V}_s+Ay)\partial_x$ is the convective derivative. Using the readily derived commutation relations,

$$[\mathcal{D}_s^n,\partial_y]=-An\mathcal{D}_s^{n-1}\partial_x, \tag{5a}$$

$$[\mathcal{D}_s^n,\Delta]=-2An\mathcal{D}_s^{n-1}\partial_x\partial_y-A^2n(n-1)\mathcal{D}_s^{n-2}\partial_x^2, \tag{5b}$$

and manipulating Eqs. (3) and (4), we obtain

$$\mathcal{D}_1^2n_1+\omega_1^2n_1=(m_2/m_1)\omega_2^2n_2+2AN_1\partial_xV_{1_y}, \tag{6a}$$

$$\mathcal{D}_2^2n_2+\omega_2^2n_2=(m_1/m_2)\omega_1^2n_1+2AN_2\partial_xV_{2_y}, \tag{6b}$$

where $\omega_s\equiv(4\pi q_s^2N_s/m_s)^{1/2}$ is the plasma frequency. Further manipulation yields

$$\mathcal{D}_s\{N_s[(\partial_x^2+\partial_z^2)V_{s_y}-\partial_y(\partial_xV_{s_x}+\partial_zV_{s_z})]+A\partial_xn_s\}=0, \tag{7a}$$

or equivalently (after taking the x derivative and rearranging terms)

$$\mathcal{D}_s\{N_s\Delta V_{s_y}+\mathcal{D}_s\partial_yn_s+2A\partial_xn_s\}=0, \tag{7b}$$

implying that the quantities in curly brackets remain constant along the flow. The latter expressions, coupled with Eqs. (5a) and (5b), help convert Eqs. (6a) and (6b) to

$$\Delta\mathcal{D}_1^3m_1n_1+\mathcal{D}_1\Delta\{m_1\omega_1^2n_1-m_2\omega_2^2n_2\}=0, \tag{8a}$$

$$\Delta\mathcal{D}_2^3m_2n_2+\mathcal{D}_2\Delta\{m_2\omega_2^2n_2-m_1\omega_1^2n_1\}=0. \tag{8b}$$

Since $m_1\omega_1^2n_1-m_2\omega_2^2n_2=4\pi q_1N_1(q_1n_1+q_2n_2)=-q_1N_1\Delta\varphi$, Eqs. (8a) and (8b) can be rewritten in the remarkably simple form:

$$\Delta\mathcal{D}_1^3(q_1n_1)+\omega_1^2\mathcal{D}_1\Delta(q_1n_1+q_2n_2)=0, \tag{9a}$$

$$\Delta\mathcal{D}_2^3(q_2n_2)+\omega_2^2\mathcal{D}_2\Delta(q_1n_1+q_2n_2)=0. \tag{9b}$$

This pair of *exact* partial differential equations constitutes the mathematical formulation of the problem.

In order to initiate the standard nonmodal analysis [14,21], we must transform to the moving frame. This is achieved by the substitutions $x'=x-(\mathcal{V}_1+Ay)t$, $y'=y$, $z'=z$, $t'=t$ with the corresponding change of operators $[\Delta\mathcal{V}\equiv\mathcal{V}_2-\mathcal{V}_1]$: $\mathcal{D}_1=\partial_{t'}$, $\mathcal{D}_2=\partial_{t'}+\Delta\mathcal{V}\partial_{x'}$, $\partial_x=\partial_{x'}$, $\partial_y=\partial_{y'}-At\partial_{x'}$, and $\partial_z=\partial_{z'}$. The Fourier transform in the new spatial variables, $F=\int dk_{x'}dk_{y'}dk_{z'}\hat{F}(k_{x'},k_{y'},k_{z'},t)\exp[i(k_{x'}x'+k_{y'}y'+k_{z'}z')]$, now converts Eqs. (2)–(4) to a set of first order, ordinary differential equations (ODE's) for the evolution of the spatial Fourier harmonics (SFH). In terms of dimensionless quantities, $D_1\equiv i\hat{n}_1/N_1$, $D_2\equiv i\hat{n}_2/N_2$, $\beta_0\equiv k_{y'}/k_{x'}$, $\gamma\equiv k_{z'}/k_{x'}$, $R\equiv A/ck_{x'}$, $\tau\equiv ck_{x'}t'$, $\beta(\tau)\equiv\beta_0-R\tau$, $\vec{U}_s\equiv\hat{V}_s/c$, $\varepsilon\equiv\Delta\mathcal{V}/c$, $\delta\equiv q_2m_1/q_1m_2$, $\Phi\equiv i|q_1|\hat{\varphi}/m_1c^2$, $W_s\equiv\omega_s/ck_{x'}$, the original equations become

$$[1+\beta^2(\tau)+\gamma^2]\Phi=W_1^2[D_1-D_2], \tag{10}$$

$$\partial_\tau U_{1_x}+RU_{1_y}=-\Phi, \tag{11a}$$

$$\partial_\tau U_{1_y}=-\beta(\tau)\Phi, \tag{11b}$$

$$\partial_\tau U_{1_z}=-\gamma\Phi, \tag{11c}$$

$$(\partial_\tau+i\varepsilon)U_{2_x}+RU_{2_y}=-\delta\Phi, \tag{12a}$$

$$(\partial_\tau+i\varepsilon)U_{2_y}=-\delta\beta(\tau)\Phi, \tag{12b}$$

$$(\partial_\tau+i\varepsilon)U_{2_z}=-\delta\gamma\Phi, \tag{12c}$$

$$\partial_\tau D_1=U_{1_x}+\beta(\tau)U_{1_y}+\gamma U_{1_z}, \tag{13}$$

$$(\partial_\tau+i\varepsilon)D_2=U_{2_x}+\beta(\tau)U_{2_y}+\gamma U_{2_z}. \tag{14}$$

In these variables, R is the effective measure of the shear strength and $\beta(\tau)$ and γ denote normalized wave vectors in the directions transverse to the flow. The integrals of the flow (associated with the operators $\mathcal{D}_s$), given by Eq. (7), reduce to the following pair of algebraic relations:

$$(1+\gamma^2)U_{1_y}-\beta(\tau)(U_{1_x}+\gamma U_{1_z})=RD_1+C_1, \tag{15a}$$

$$(1+\gamma^2)U_{2y}-\beta(\tau)(U_{2x}+\gamma U_{2z})=RD_2+C_2e^{-i\varepsilon\tau}, \tag{15b}$$

where C_1 and C_2 are some constants.

The dimensionless manifestation of Eqs. (6a) and (6b) are the following second order ODE's:

$$\partial_\tau^2D_1+W_1^2D_1=W_1^2D_2-2RU_{1y}, \tag{16a}$$

$$(\partial_\tau+i\varepsilon)^2D_2+W_2^2D_2=W_2^2D_1-2RU_{2y}. \tag{16b}$$

Combining Eqs. (11)–(16), we derive

$$\partial_\tau^2P+\left[W_1^2+\frac{3R^2(1+\gamma^2)}{\mathcal{K}^4(\tau)}\right]P=W_1^2e^{-i\varepsilon\tau}Q-\frac{2RC_1}{\mathcal{K}^3(\tau)}, \tag{17a}$$

$$\partial_\tau^2Q+\left[W_2^2+\frac{3R^2(1+\gamma^2)}{\mathcal{K}^4(\tau)}\right]Q=W_2^2e^{i\varepsilon\tau}P-\frac{2RC_2}{\mathcal{K}^3(\tau)}, \tag{17b}$$

where $\mathcal{K}(\tau)\equiv\sqrt{1+\beta^2(\tau)+\gamma^2}$, $P\equiv D_1\mathcal{K}^{-1}(\tau)$, and $Q\equiv D_2e^{i\varepsilon\tau}\mathcal{K}^{-1}(\tau)$.

These equations constitute a basic set of ODE's describing the temporal evolution of SFH in the two-component, cold plasma shear flow. Evidently in the shearless ($R=0$, $C_s=0$) limit the equations encompass the traditional, simple electrostatic plasma physics: the plasma oscillations and (under certain well-known conditions) the two stream instability. One can easily surmise that additional novelty will result from the interplay of standard plasma effects and the velocity shear induced effects. These will certainly include (a) the variation of the wave number of each SFH in time (due to the effect of the shearing background on the wave crest); (b) the appearance of algebraic, nonexponentially evolving solutions; and (c) interaction of wavelike and vortexlike solutions with each other.

Mathematically the first effect is contained in the time dependence of the function $\mathcal{K}(\tau)$. The appearance of the nonexponentially evolving, vortex solutions (the main subject of the present paper), on the other hand, is connected with the existence of the inhomogeneous terms (terms proportional to C_1 and C_2) in Eq. (17). Notice that the terms proportional to C_1 and C_2 will *vanish* if the shear parameter R is zero. Thus any effect emerging from the inhomogeneous terms in Eq. (17) is naturally induced by the velocity shear.

III. ''SHEAR LANGMUIR'' VORTEXES

The basic properties of the vortical solutions are best delineated in a model in which there is no mutual streaming ($\mathcal{V}_1=\mathcal{V}_2=\mathcal{V}$, $\varepsilon=0$) of the species. The initial system (11)–(17), then, may be elegantly rewritten through one-fluid variables: the perturbed charge density $\varrho\equiv q_1n_1+q_2n_2$ and the current density $\mathbf{J}\equiv q_1N_1\mathbf{V}_1+q_2N_2\mathbf{V}_2$. The result is

$$j_x^{(1)}=-Rj_y-(W/\mathcal{K})^2D, \tag{18a}$$

$$j_y^{(1)}=-(W/\mathcal{K})^2\beta D, \tag{18b}$$

$$j_z^{(1)}=-(W/\mathcal{K})^2\gamma D, \tag{18c}$$

$$D^{(1)}=j_x+\beta j_y+\gamma j_z, \tag{19}$$

where $W\equiv\omega_p/k_{x_1}c$, $D\equiv i\hat{\varrho}/c|q_1|N_1$, $\mathbf{j}\equiv\hat{\mathbf{J}}/c|q_1|\mathcal{N}_1$, and $\omega_p^2\equiv\omega_1^2+\omega_2^2$. Note that the translation of the algebraic relations (15) in the one-fluid variables amounts to $[\mathcal{C}\equiv C_1-C_2]$:

$$(1+\gamma^2)j_y-\beta(j_x+\gamma j_z)=RD+\mathcal{C}. \tag{20}$$

It is also straightforward to see that the ''energy''

$$\mathcal{E}\equiv\frac{1}{2}\left[|j_x|^2+|j_y|^2+|j_z|^2+\frac{W^2}{\mathcal{K}^2}|D|^2\right], \tag{21}$$

of the SFH varies as

$$\mathcal{E}^{(1)}=-R\left(j_x j_y-\frac{W^2\beta}{\mathcal{K}^4}D^2\right), \tag{22}$$

implying that in the shearless ($R=0$) limit, the energy of each SFH is a conserved quantity.

From Eqs. (18) and (19) it easily follows that $D^{(2)}+W^2D+2Rj_y=0$, and taking into account (20) for $Y\equiv D/\mathcal{K}$, we get

$$Y^{(2)}+\left[W^2+\frac{3R^2(1+\gamma^2)}{\mathcal{K}^4}\right]Y=-\frac{2R\mathcal{C}}{\mathcal{K}^3}, \tag{23}$$

which could also be derived from Eq. (17) identifying $Y\equiv P-Q$. Further analysis will be dedicated to the solution of this equation. Note that all physical variables characterizing the perturbations are readily expressed in terms of $Y(\tau)$ and $Y^{(1)}(\tau)$.

It should be noted that Eq. (23) is somewhat analogous with equations that govern shear-induced evolution of 2D [14] and 3D [17] acoustic perturbations in hydrodynamic parallel shear flows. It is also similar to the evolution equation for the electrostatic ion perturbations in the plasma shear flow [26]. Below we shall exploit this analogy by harnessing the mathematical methods used in Refs. [14], [17], and [26].

In the shearless limit, Eq. (23) must and does describe elementary plasma oscillations. With nonzero shear, the oscillations are modified and become dispersive. Besides, and this is more important, velocity shear causes the appearance of a *new class* of solutions driven by the inhomogeneous term in Eq. (23). General solution of the equation is the sum of the *particular* solution and the *general* solution of the corresponding homogeneous equation (with $\mathcal{C}=0$). In other words, Eq. (23) contains the seeds of two different modes of plasma collective behavior: (a) Plasma (Langmuir) oscillations, modified by the presence of the velocity shear—$\mathcal{C}=0$; (b) Aperiodic vortex perturbations—$\mathcal{C}\neq 0$.

This classification is strongly justified for flows with $R\ll 1$, while for $R\simeq 1$ it becomes quite ill defined. For $R\ll 1$, a particular solution of Eq. (23), proportional to the inhomogeneity parameter $\mathcal{C}$, may be readily found. Introducing the auxiliary notation $\mathcal{Y}\equiv W^2Y/2R\mathcal{C}$, $\nu\equiv R/W$, and remembering that $\beta(\tau)\equiv\beta_0-R\tau$, we can reduce Eq. (23) to

$$\nu^2\frac{\partial^2\mathcal{Y}}{\partial\beta^2}+\left[1+\frac{3(1+\gamma^2)\nu^2}{\mathcal{K}^4}\right]\mathcal{Y}+\frac{1}{\mathcal{K}^3}=0. \tag{24}$$

The parameter $\nu\sim A/\omega_p$ may take low ($\nu\ll 1$) or moderate ($\nu\lesssim 1$) values in different cases of practical (astrophysical or laboratory) importance. The latter range, for example, may be expected in astrophysical objects characterized by very high-energy processes [28]. Since the linear dynamics of SLV for the low and moderate shear rates is so different, we will analyze these cases separately in the next section. For low shear, approximate analytic treatment gives a complete description of the phenomena: smallness of ν allows us to set up a systematic nonsingular perturbation theory. The particular solution of Eq. (23) is given by the series [29,14,17,26]

$$\mathcal{Y}=\sum_{n=0}^{\infty}\nu^{2n}\mathcal{Y}_n, \tag{25a}$$

$$\mathcal{Y}_0(\beta)=-(1+\beta^2+\gamma^2)^{-3/2}, \tag{25b}$$

$$\mathcal{Y}_n(\beta)=-\left[\frac{\partial^2\mathcal{Y}_{n-1}}{\partial\beta^2}+\frac{3(1+\gamma^2)\mathcal{Y}_{n-1}}{(1+\beta^2+\gamma^2)^2}\right], \tag{25c}$$

where the leading order solution $\mathcal{Y}_0$ contains most of the essential features of the shear-driven Langmuir vortex.

The vortex solution looks a bit simpler in the (θ,ϕ) notation: θ is the angle between the $\mathbf{k}(\tau)$ vector and the Y axis; and ϕ measures the angle between the X axis and the projection of $\mathbf{k}$ onto X-Z plane. Shear-induced drift of wave vectors, as we have seen above, causes temporal variation of k_y, so that θ varies with time, while ϕ remains constant. In particular, $\tan\phi=\gamma=$const and $\cos\theta(\tau)=\beta(\tau)/\mathcal{K}(\tau)$. One can notice that adopting $\theta(\tau)$ as new independent variable, introducing new dimensionless parameter $\alpha\equiv R\cos\phi/W=\nu\cos\phi$, and a new function $\Psi\equiv(W/2\alpha\mathcal{C})\sec^2\phi\sin\theta Y$, we can rewrite Eq. (23) in the form of inhomogeneous Hill's differential equation [30]: $\alpha^2\partial_\theta^2\Psi+[4\alpha^2+\sin^{-4}\theta]\Psi+1=0$.

The physical quantities associated with the leading order SLV solution $\mathcal{Y}_0$ are, now, expressed as

$$RD=-2\alpha^2\mathcal{C}\sin^2\theta, \tag{26a}$$

$$j_x=\mathcal{C}\cos^3\phi[\tan^2\phi\ \theta+\sin\theta\cos\theta], \tag{26b}$$

$$j_y=\mathcal{C}\cos^2\phi\sin^2\theta, \tag{26c}$$

$$j_z=\mathcal{C}\sin\phi\cos^2\phi[\theta-\sin\theta\cos\theta], \tag{26d}$$

$$\mathcal{E}\simeq\frac{\mathcal{C}^2}{2}\cos^4\phi[\sin^2\theta+\tan^2\phi\times\theta^2]. \tag{26e}$$

Note that in the latter expression, the contribution of the last term (D^2 term) in the energy expression (21) is neglected, because direct calculation shows that it is at least α^2 times less than the other terms.

The same solutions, exposed in the initial notation, are

$$D=-\left(\frac{2R\mathcal{C}}{W^2}\right)\frac{1}{1+\gamma^2+\beta^2}, \tag{27a}$$

$$j_x=-\left(\frac{\mathcal{C}}{(1+\gamma^2)^{3/2}}\right)\left[\gamma^2\mathrm{acot}\left(\frac{\beta}{\sqrt{1+\gamma^2}}\right)+\frac{\beta\sqrt{1+\gamma^2}}{1+\gamma^2+\beta^2}\right], \tag{27b}$$

$$j_y=\frac{\mathcal{C}}{1+\gamma^2+\beta^2}, \tag{27c}$$

$$j_z=-\left(\frac{\mathcal{C}\gamma}{(1+\gamma^2)^{3/2}}\right)\left[\mathrm{acot}\left(\frac{\beta}{\sqrt{1+\gamma^2}}\right)-\frac{\beta\sqrt{1+\gamma^2}}{1+\gamma^2+\beta^2}\right], \tag{27d}$$

$$\mathcal{E}\simeq\frac{\mathcal{C}^2}{2(1+\gamma^2)^2}\left[\frac{1+\gamma^2}{1+\beta^2+\gamma^2}+\gamma^2\mathrm{acot}^2\left(\frac{\beta}{\sqrt{1+\gamma^2}}\right)\right]. \tag{27e}$$

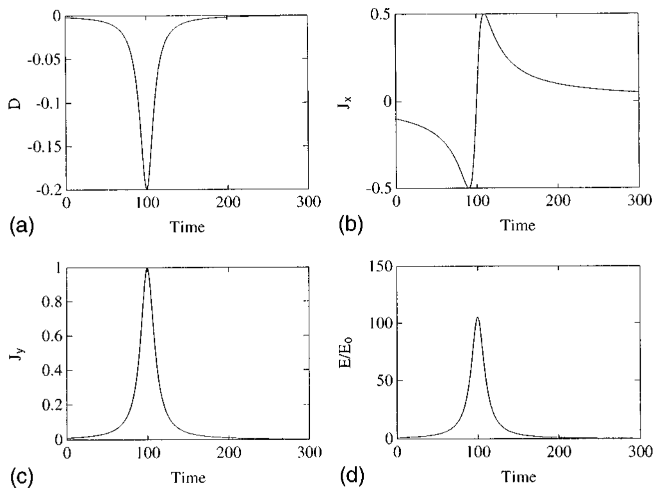

FIG. 1. Leading order solutions of $D(\tau)$, $J_x(\tau)$, $J_y(\tau)$, and $\mathcal{E}(\tau)/\mathcal{E}_0$ for 2D SLV when $\beta_0=10$, $R=0.1$, $W=1$, and $\mathcal{C}=1$.

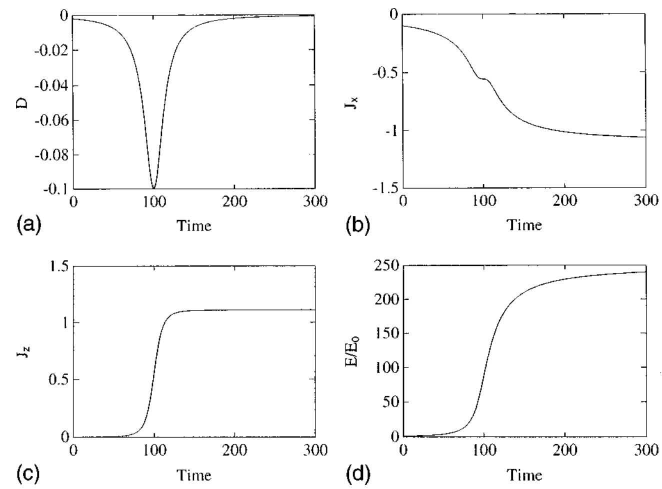

FIG. 2. Leading order solutions of $D(\tau)$, $J_x(\tau)$, $J_z(\tau)$, and $\mathcal{E}(\tau)/\mathcal{E}_0$ for 3D SLV when $\beta_0=10$, $\gamma=1$, $R=0.1$, $W=1$, and $\mathcal{C}=1$.

IV. DISCUSSION

We are now ready to discuss the main features of the analytic solutions for shear Langmuir vortexes. Later, we shall also treat moderate and high shear cases and show that in this regime SLV solutions eventually acquire wavelike features. This phenomenon, which is rather similar to the one recently found [27] for the simple example of an unbounded, compressible plane Couette flow of a neutral fluid, is described phenomenologically as the conversion of SLV into Langmuir oscillations.

A. Low-shear limit

It is convenient to consider the two-dimensional (2D) and the three-dimensional (3D) cases separately.

(i) *2D perturbations*, $\gamma=0$ $(\phi=0)$–these perturbations lie in the XOY plane. General SLV solutions reduce to the following simple expressions:

$$D=-(2R\mathcal{C}/W^2)(1+\beta^2)^{-1}, \tag{28a}$$

$$j_x=-\mathcal{C}\beta(1+\beta^2)^{-1}, \tag{28b}$$

$$j_y=\mathcal{C}(1+\beta^2)^{-1}, \tag{28c}$$

$$j_y=(\mathcal{C}^2/2)(1+\beta^2)^{-1}. \tag{28d}$$

Plots representing temporal evolution of these solutions are presented in Figs. 1(a)–1(d), respectively. The figures are drawn for the following values of the parameters: $\beta_0=10$, $R=0.1$, $W=1$, $\mathcal{C}=1$. The energy is normalized on its initial value in order to highlight the rate of transient increase. In particular, the transient increase in energy takes place if initially $k_{y_1}/k_{x_1}>0$ $(\beta_0>0)$, and it occurs around the time $\tau_*\equiv\beta_0/R$ when $\beta(\tau)$ tends to zero, and $(1+\beta^2)^{1/2}$ attains its minimum value equal to one. Geometrically, it is the very moment of time when the wave number vector $\mathbf{k}$ becomes perpendicular to Y axis. The transient rate of the energy increase crucially depends on the *initial orientation* of the perturbation wave vector in space. In fact, we find from $\mathcal{E}_{\max}/\mathcal{E}_0=(1+\beta_0^2)=1/\sin^2\theta_0$, that SLV's with large values of β_0 are the only ones that will show a substantial transient increase. Geometrically, these are the perturbations that are, initially, almost aligned with the Y axis.

(ii) *3D perturbations*: These are considerably more complex and rich. Here, the trigonometric representation of solutions is especially convenient for analyzing the SLV features. For clarity of exposition, let us consider a SFH with positive initial values k_x, k_y, and k_z. In the course of evolution, the angle θ traverses, monotonically, the range $[\theta_0, \pi]$, acquiring the value $\theta_{\tau_*}=\pi/2$ at $\tau=\tau_*\equiv\beta_0/R$. And as $\tau\rightarrow\infty$, the asymptotic value $\theta_\infty=\pi$ is reached.

From Eqs. (26), we may deduce that the behavior of the charge density, and of the y component of the current density still remains ''transient'' just like the 2D perturbations; the extra dimension does not affect them much. However, the situation with the x and z components of the current density [normal to the ''shear'' (Y) axis] is crucially different; they acquire ''nontransient,'' increasing terms (proportional to the angle θ), which tend eventually to saturation. This phenomenon is illustrated by Figs. 2(a)–2(d), where $D(\tau)/D(0)$, $j_x(\tau)/j_x(0)$, $j_z(\tau)/j_z(0)$, and $\mathcal{E}(\tau)/\mathcal{E}_0$ functions are plotted for the same set of parameters as Fig. 1, but with $\gamma=1$.

Using Eq. (26) it is easy to write the ratio of the asymptotic and initial value of the SLV energy:

$$\frac{\mathcal{E}_\infty}{\mathcal{E}_0}=\frac{\pi^2\tan^2\phi}{\sin^2\phi_0+\tan^2\phi\ \theta_0^2}, \tag{29}$$

which reduces to $\mathcal{E}_\infty/\mathcal{E}_0\simeq\sin^2\phi(\pi/\theta_0)^2$ when $\theta_0\ll1$. This estimate implies that the asymptotic energy of SLV (which is equal to zero for 2D perturbations) may be either less or greater than its initial energy. Naturally, for the quasi-2D SLV's, with very small values of ϕ, $\mathcal{E}_\infty<\mathcal{E}_0$ even when $\theta_0\ll\pi$. However, for

$$\phi>\phi_t\equiv\arctan\left(\frac{\sin\ \theta_0}{\sqrt{\pi^2-\theta_0^2}}\right), \tag{30}$$

where ϕ_t serves as some threshold value of ϕ, the asymptotic energy of the SLV becomes greater than its initial energy. In other words, the SLV's, for which the initial spa-

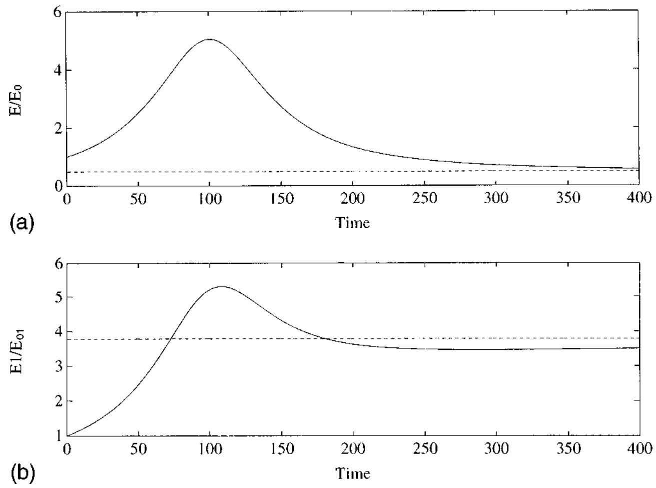

FIG. 3. Temporal evolution of $\mathcal{E}(\tau)/\mathcal{E}_0$ for leading order 3D SLV solution when $\beta_0=2$, $R=0.02$, $W=1$, and $\mathcal{C}=1$, while $\gamma=0.1$ (a) and $\gamma=0.2$ (b). Dashed horizontal line shows asymptotic value of the perturbation energy $\mathcal{E}_\infty$.

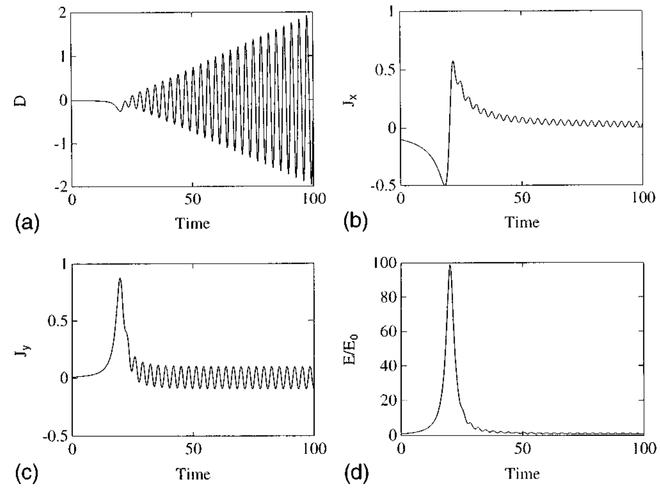

FIG. 5. Numerical solutions of the (18)–(19) system for 2D SLV, when $\beta_0=10$, $W=2$, while $R=0.5$ ($\tau_*=20$). The graphs display how initially purely vortical solution acquires wavelike features at $\tau>\tau_*$.

tial orientation is such that condition (30) is satisfied, become *algebraically unstable*: they can extract energy from the mean flow not only *transiently but also asymptotically*.

The plots, illustrating these interesting properties of 3D SLV's are presented in Fig. 3 and Fig. 4. In particular, Fig. 3(a) is drawn for $W=1$, $\mathcal{C}=1$, $\beta_0=2$, $R=0.02$, and $\gamma=0.1$. For this sample one can easily check that $\phi\simeq5.7°$ and $\phi_t\simeq8.0°$, so that ϕ is below the threshold for asymptotic growth; the ratio of the asymptotic to the initial energy comes to be $\mathcal{E}_\infty/\mathcal{E}_0\simeq0.5$. Figure 3(a) shows explicitly that the SLV energy increases transiently, attains its maximum (transient) value at $\tau=\tau_*$, and decreases towards its asymptotic value $\mathcal{E}_\infty$, which is *less* than the initial value. This behavior of the quasi-2D SLV is reminiscent of the 2D case.

When γ is slightly larger ($\gamma=0.2$), the situation changes quite drastically. With $\phi\simeq16.7°$ greater than the threshold $\phi_t\simeq8.3°$, the asymptotic energy overtakes the initial energy, $\mathcal{E}_\infty/\mathcal{E}_0\simeq4$. Figure 3(b) displays the entire temporal evolution: the initial transient energy increase, leading to the maximum (transient) value at $\tau=\tau_*$, is followed by a decrease towards the asymptotic value $\mathcal{E}_\infty$. But now the asymptotic value is a few times larger than the initial one implying a considerable departure from the 2D picture.

The same tendency is more clearly pronounced in Fig. 4(a) with parameters: $\beta_0=10$, $R=0.1$, and $\gamma=0.3$ (W and $\mathcal{C}$ are taken the same as above). In this case $\phi\simeq16.7°$ again but $\phi_t\simeq1.9°$, so that ϕ is much larger than the threshold value. As a consequence, the ratio of the asymptotic to the initial energy, $\mathcal{E}_\infty/\mathcal{E}_0\simeq75$, becomes rather large. We see that the energy, after initial transient increase, decreases, passes through a broad minimum and begins to increase again tending to its second (asymptotic) maximum value $\mathcal{E}_\infty$.

This complex evolutionary behavior of the energy gives way to a monotonic increase for larger values of γ. This is illustrated in Fig. 4(b), for which $\gamma=1$, $\beta_0=10$, $R=0.1$ (W, $\mathcal{C}$ are the same again). In this case $\phi\simeq45.0°$, $\phi_t\simeq2.5°$, and the ratio of the asymptotic to the initial energy is very large: $\mathcal{E}_\infty/\mathcal{E}_0\simeq250$. Since the asymptotic value is larger than the maximum transient value, the transient increase is completely overwhelmed by the asymptotic increase and the energy of the SLV increases monotonically tending to saturate at the value $\mathcal{E}_\infty$.

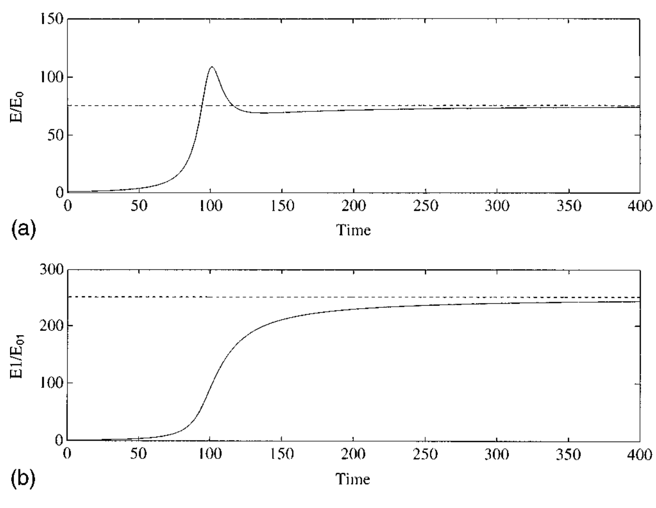

FIG. 4. Temporal evolution of $\mathcal{E}(\tau)/\mathcal{E}_0$ for leading order 3D SLV solution when $\beta_0=10$, $R=0.1$, $W=1$, and $\mathcal{C}=1$ while $\gamma=0.3$ (a) and $\gamma=1$ (b). Dashed horizontal line shows asymptotic value of the perturbation energy $\mathcal{E}_\infty$.

B. Moderate and high shear limit

For moderate and high values of the shear parameter, yet another interesting phenomenon appears on the scene; the *conversion of SLV into Langmuir oscillations*. This phenomenon closely resembles the recently found 'Conversion of vortex mode into the acoustic wave' [27]. Simple and generic nature of this phenomena suggests that it should exist in a wide variety of shear flows of different origin and composition. Numerical examination of our specific problem fully supports such an expectation.

For moderate and large shear, we must perform a numerical study of our basic Eqs. (18) and (19). Beginning with a 2D case, we choose the initial conditions to guarantee the

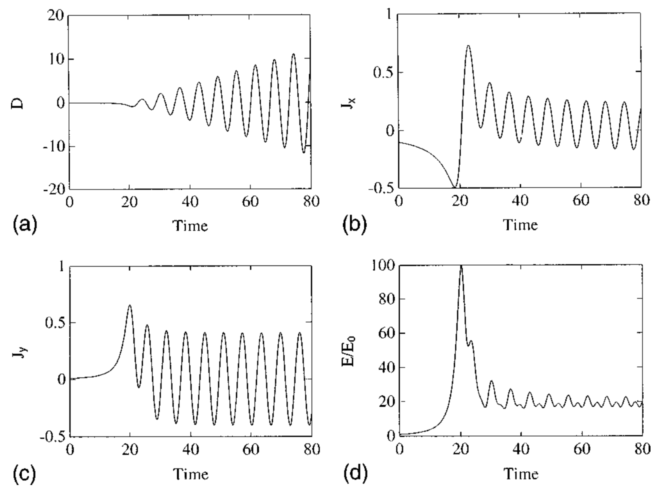

FIG. 6. Numerical solutions of the (18)–(19) system for 2D SLV, when $\beta_0=10$, $W=1$, while $R=0.4$ ($\tau_*=20$).

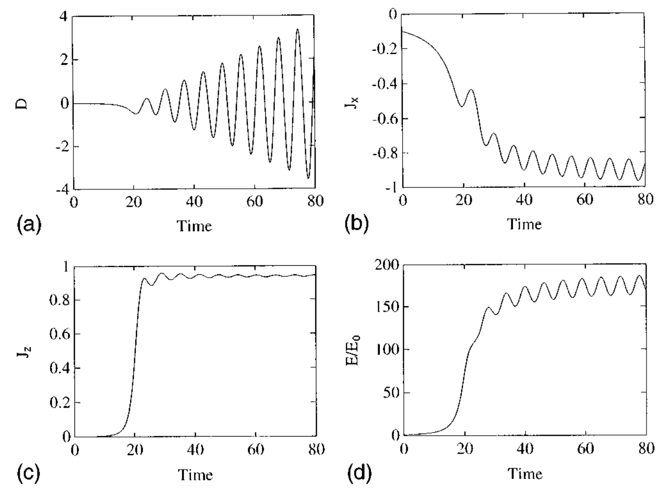

FIG. 7. Numerical solutions of the (18)–(19) system for 3D SLV, when $\beta_0=10$, $\gamma=1$, $W=1$, while $R=0.5$ ($\tau_*=20$).

excitation of a nonoscillating, pure SLV at $\tau=0$. This structure, however, does not stand the test of time. At $\tau=\tau_*$, we clearly witness the emergence of the oscillating Langmuir waves (Figs. 5 and 6). The initial inhomogeneous solution (SLV) must change to a mixture of the homogeneous (Langmuir wave) and the inhomogeneous in order to satisfy the defining equation at later times. The time evolution, therefore, imparts more and more wavelike features to the initial vortical perturbation as if the latter was undergoing mode ''conversion.'' Obviously, the wavelike features could easily become dominant at large times.

The mode ''conversion'' process is gradual (abrupt) for smaller (larger) values of the ratio $\nu\equiv R/W$. The difference in the two situations becomes apparent on comparing Figs. 5 and 6. The parameters for Fig. 5, $W=2$, $\mathcal{C}=1$, $\beta_0=10$, and $R=0.5$, lead to a small value of $\nu=0.25$, and we see [in the $D(\tau)$ plot] a gradual process, stretched in time showing how the vortex acquires wavelike features. Note that other variables [$j_x(\tau)$, $j_y(\tau)$, and $\mathcal{E}(\tau)/\mathcal{E}_0$] also exhibit small but perceptible changes in their structure.

For the plots of Fig. 6, we double the value of ν by choosing $W=1$. Unlike the previous case, we now observe an ''abrupt'' appearance of wavelike features [27]. In this case the appearance of wavelike behavior is clearly visible for all the variables.

Evolution of SLV's is different in the 3D case. Numerical calculations carried for $W=1$, $\mathcal{C}=1$, $\beta_0=10$, $\gamma=1$, and $R=0.5$ are presented in Fig. 7. The plots show that here also we find, as the 2D case, the emergence of oscillations from the vortex. But, the asymptotic result, unlike the 2D case, contains both the Langmuir oscillations and the SLV. This should be expected, since in the 3D case, the SLV's are able to extract the mean flow energy not only transiently, but also asymptotically (see, for comparison, Fig. 4).

V. CONCLUSION

In this paper, we have derived and investigated a mode of nonperiodic collective behavior evoked by the presence of the kinematic shear in plasma flows. The shear Langmuir vortexes have rather unique and interesting characteristics, not the least of which is the ability of the 3D SLV to extract energy from the mean flow even asymptotically. There are very few known cases in which a simple linear system displays an asymptotic algebraic instability. Although it is not easy to pinpoint, at once, the potential applications of SLV's, it is reasonable to expect that these and analogous structures (investigation of which seems to be similar to this one) should lead to significant and measurable effects in various laboratory, fusion, geophysical, and astrophysical sheared plasma flows. One can expect that SLV's, alone, or through their interaction with the Langmuir waves could seriously change the high frequency behavior of sheared plasmas. It is possible, for example, that transiently evolving 2D, and especially the ''unstable'' 3D SLV, could cause a new kind of *anomalous resistivity*. These structures may also play a credible role in the subcritical onset of turbulence in plasma shear flows.

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